A MONTE CARLO DISPERSION ANALYSIS OF A ROCKET FLIGHT SIMULATION SOFTWARE

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Abstract: A Monte Carlo dispersion analysis has been completed on a medium range solid propellant rocket simulation software. This analysis has been carried out to find the optimum values of the rocket fincant angle and spin motor torque for the best impact point error, and the probability of flight-to-target success. The simulation is develeoped based on rotating earth equations of motion and has many components. This paper describes the methods used to accomplish the Monte Carlo analysis and gives an overview of the processes used in the implementation of the dispersions. Selected results from 70000 Monte Carlo runs are presented with suggestions for the values of the desired parametres.

Keywords: Flight Simulation, Stochastic, Monte Carlo, Dispersion Analysis, Rocket

INTRODUCTION:

If all characteristics of a rocket, together with atmospheric conditions are exactly equal to a set of predicted values, the rocket will fly on a known trajectory and hits a target point. This trajectory is called nominal trajectory. In practice, there are always some differences between the real and predicted values. These are mainly due to manufacturing, measurement and atmospheric modeling errors. These differences make the rocket not to fly exactly on its nominal trajectory, and to hit a target. Therefore, there are always some errors between the positions of a desired and a real impact point. Estimation of these errors is very important from the operational point of view. Also, investigation of the error sources and their effects can help a rocket designer to optimize design parameters for the lowest impact point error.

In this study, a Monte Carlo dispersion analysis has been completed on flight simulation software of a rocket to investigate its impact point error. The rocket is a solid propellant medium range type with 320-km range, 9-m length, 0.5-m diameter and a maximum weight of 3500 kg. Four cross type stabilizer fins and a spin motor provide the rocket static stability. The rocket six-degree-of-freedom simulation was used to repeatedly fly a near nominal trajectory. This simulation software has been developed based on the rotating earth equations of motion. It has many components such as, aerodynamics, mass properties, equations of motion, atmospheric model, wind model at different altitudes and a gravity model. No intervention was required to simulate a complete trajectory because there is no control on a rocket after it is launched. This allows multiple runs to be directly compared.

The Monte Carlo method of dispersion analysis uses a given system model (in this case, the rocket flight mathematical model) and introduces statistical uncertainties on as many of the individual parameters as practical. In this work, a set of forty-one parameters is selected in different categories including Aerodynamics, Propulsion, Atmosphere and Wind, Mass and Inertia, Dimension and Launching. A uniform distribution of uncertainties around the nominal values of each parameter is considered. The range of magnitudes of uncertainties are defined based on a set of known observations and a first step individual error analysis. Uniformly distributed random values in the defined ranges were selected and applied to the simulation parameter. Each Monte Carlo simulation run had different random variation of the dispersions.

The number of Monte Carlo runs containing uncertainty combinations that result in failure to complete a normal flight were identified. Thus, the probability of flight-to-target success was established. Although, establishing the probability of flight-to-target success was one of the primary goals of this analysis, other objectives such as system validation also were accomplished. Completing, the Monte Carlo analysis also allowed for the identification of weaknesses in rocket design and margins in specific rocket parameter.

The objective of this report is to demonstrate how Monte Carlo simulation analysis can be used to identify and analyze the trajectory problems for a rocket and provide some preliminary results. Results

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are presented for 70000 Monte Carlo runs done in this analysis in the form of dispersion plots of different parameters including maximum angular speed, flight time, maximum speed, maximum angle of attack, range error, directional error and radial error.

DISPERSION MODELS:

The dispersions used in the Monte Carlo simulation has been applied to the rocket dynamics and external enviroment models. The models modified in the rocket simulation to include dispersion capabilities were the aerodynamics, mass properties, propulsion, atmospheric and launching models. Table 1 shows a list of fourty-one uncertainty parameters that have been used in this work. It is tried to consider all the important parameters except the aerodynamic coefficients. A similar study has already been carried out to investigate the effect of the aerodynamic coefficients uncertainties on the rocket impact point error when all other parametres were in their nominal values [Sarikhani and Roshanaiyan, 2002]. Therefore, in the present work, for the sake of computational time reduction, efforts focused to find out the errors associated with the other parameter uncertainties.

Table 1: Uncertainty parameters and ranges

	Parameter definition	**Uncertainty range**	**Unit**
1	Launching pitch angle	[-0.3 0.3]	deg
2	Launching yaw angle	[-0.5 0.5]	deg
3	Fuel burnning time	[-1.0 1.0]	sec
4	Fuel mass	[-1.0 1.0]	%
5	Rocket gross weight	[-0.75 0.75]	%
6	Angular thrust vector deviation in xz plane	[-0.3 0.3]	deg
7	Angular thrust vector deviation in xy plane	[-0.3 0.3]	deg
8	Linear thrust vector deviation in body x direction	[-20 20]	mm
9	Linear thrust vector deviation in body y direction	[-10 10]	mm
10	Linear thrust vector deviation in body z direction	[-10 10]	mm
11	Thrust	[-1.0 1.0]	%
12	Rocket lenght	[-50 50]	mm
13	Rocket diameter	[-2 2]	mm
14	Moment of inertia Ixx, (with fuel)	[-2 2]	%
15	Moment of inertia Ixx, (without fuel)	[-2 2]	%
16	Moment of inertia Iyy, (with fuel)	[-2 2]	%
17	Moment of inertia Iyy, (without fuel)	[-2 2]	%
18	Center of mass position, (with fuel)	[-20 20]	mm
19	Center of mass position, (without fuel)	[-20 20]	mm
20	Air density	[-5 5]	%
21	Wind speed, zero altitude	[-2 2]	m/s
22	Wind direction, zero altitude	[-2 2]	deg
23	Wind speed, 1km altitude	[-5 5]	m/s
24	Wind direction, 1km altitude	[-5 5]	deg
25	Wind speed, 2km altitude	[-5 5]	m/s
26	Wind direction, 2km altitude	[-5 5]	deg
27	Wind speed, 5km altitude	[-10 10]	m/s
28	Wind direction, 5km altitude	[-5 5]	deg
29	Wind speed, 10km altitude	[-10 10]	m/s
30	Wind direction, 10km altitude	[-5 5]	deg
31	Jet force damping coefficient	[-0.1 0.1]	ton.m/s
32	Jet moment damping coefficient	[-0.2 0.2]	ton.m^2/s
33	Launcher coefficient of friction	[-15 15]	%
34	Spin motor starting time	[-5 5]	%
35	Spin motor operation time	[2 2]	%
36	Spin motor torque	[-0.0 0.0]	%
37	Izz & Iyy difference	[-0.2 0.2]	%
38	Product moment of inertia, Ixy	[-5 5]	% of Ixx
39	Product moment of inertia, Ixz	[-5 5]	% of Ixx
40	Product moment of inertia, Iyz	[-5 5]	% of Ixx
41	Fincant angle	[-0.0 0.0]	deg

The limits of uncertainties presented in Table 1 were based on a set of previously known observations and meaurements completed with a first step individual error analysis. In this individual error analysis, a range of values in the defined limits, Table 1, was given to each parameter and simulation was run several times. Using simulation results, it was possible to plot the impact point distance error vs different parameters variation. Some of these plots are shown in Figs. 1-6. The plots were then used to find a good estimation for each parameter uncertainty, so that the impact point error was in a normal practically observed range. The analysis discussed in this report tested over the entire uniform distribution mainly to ensure that most of the worst possible cases are considered, and to identify the probability of flight-to-target success.

Fig 1: The effect of thrust misalignment (e1) on dispersion

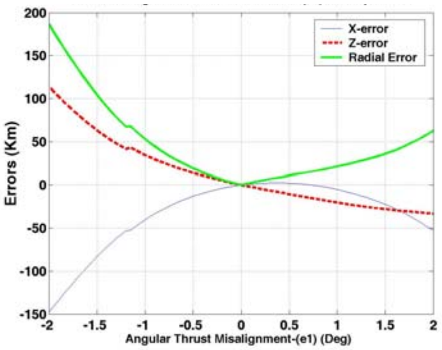


Fig 2: The effect of thrust error on dispersion

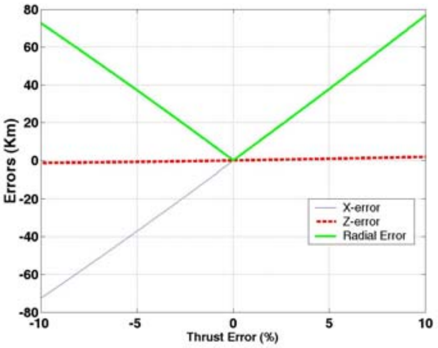


Fig 3: The effect of air density error on dispersion

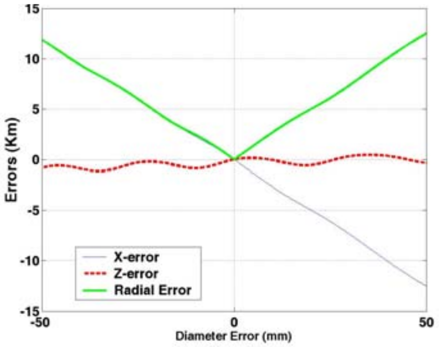


Fig 4: The effect of rocket diameter error on dispersion

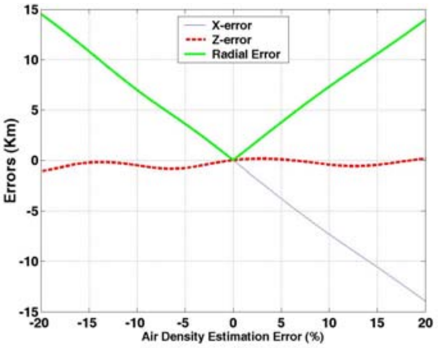


Fig 5: The effect of elevation angle error on dispersion

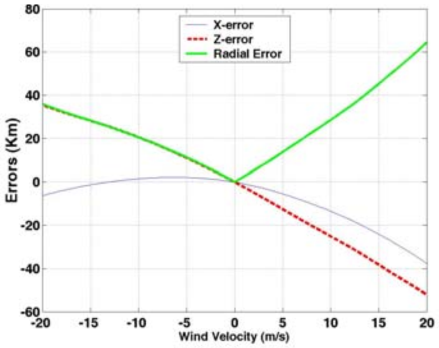
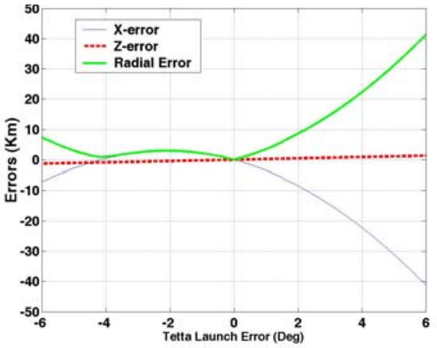


Fig 6: The effect of wind velocity at h=0 on dispersion



METHODS OF APPROACH:

Monte Carlo analysis estimates the statistics of random variables by analyzing the statistics of many trials. One important question associated with Monte Carlo analysis is determining the number of trials needed before the statistics of a variable can be estimated with reasonable accuracy. In this work, the number of Monte Carlo trials were determined based on the work previously done by [Williams, 2001], and was selected to be one thousant trials. When the desired number of runs was determined, files were generated containing all relevant dispersions. Dispersion values were randomly selected from a uniform distribution, and then stored in individual input files.

This collection of files was sequentially run from a main script, which directed the storage of relevant data. Additional scripts were written to process the data for analysis. Because each simulation run lasted approximately two minutes and was recording large amounts of data, storing the relevant data for flight without storing the entire data file generated by the simulation became necessary. For this reason, scripts were developed that took “snapshots” of the data. This snapshot process was performed on the entire data file after a run was completed. These scripts extracted the data at the beginning of each flight phase and directed the storage into separate and much smaller files. In this way, most of the data were discarded, and the process of completing many runs could be automated without exceeding memory limitations.

One of the most important reasons for the present work to be carried out was to find out the effect of the rocket fincant angle and spin motor torque on the minimum impact point error, and select the best combination of them. These are two terms which can be set relatively accurate during the rocket manufacturing time. Therefore, these two parameters were selected as control parameters in running the simulation software. A set of known discrete values for fincant angle containing –0.5, -0.2, 0.0, 0.3, 0.5, 0.7 and 1.0 deg. was selected. For each of the selected values, dispersion analysis has been carried out over a range of spin motor torque from 0 to 9000 N.M with a step of 1000 N.M. For each pair of the fincant angle and spin motor torque, 1000 simulation were run, totally 70000 times, in which all dispersions were randomly varried over a uniform distribution. For each simulation run, the maximum value of the rocket angle of attack, sideslip angle, linear and angular velocities and accelerations, dynamic pressure and attitude angles during flight were obtained and stored together with the flight time, range, directional and radial impact point errors and the random values selected automatically for the other thirty-nine parameters in Table 1.

SIMULATION RESULTS:

This section presents results for the 70000 Monte Carlo runs completed in this analysis. Some typical distribution plots of various variables are shown in Figs. 7-10. The plotted data in these figures are generated with 0.5 deg. fincant angle and maximum spin motor torque. As shown, a reasonable normal distribution of different variables is observed.

Fig 7: The freq. distrib. of flight time (Max. Spin, Fin=0.5 deg)

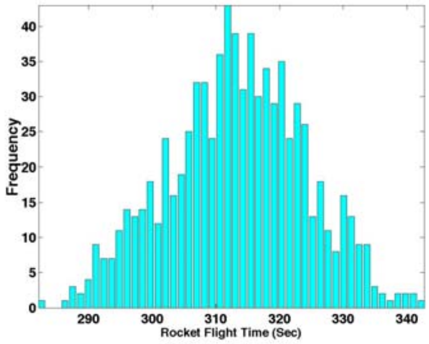


Fig 8: The freq. distrib. of “Max Wx” (Max. Spin, Fin=0.5deg.)

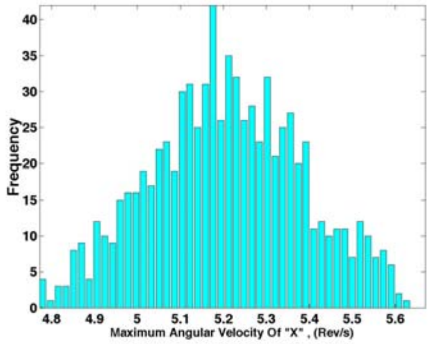


Fig 9: The freq. distrib. Of Max. A.O.A (Max Spin, Fin=0.5 deg.)

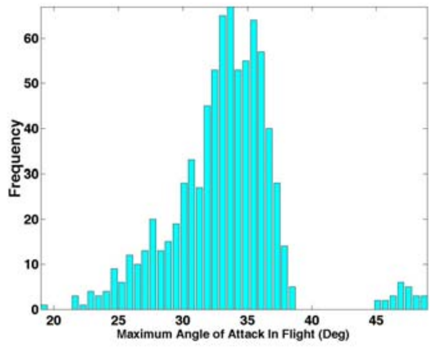
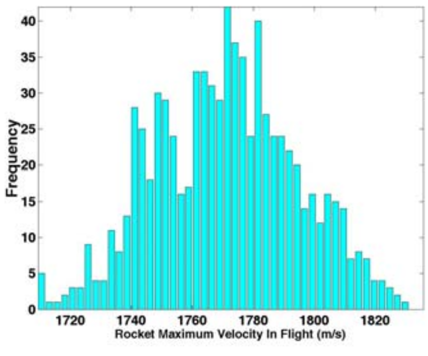


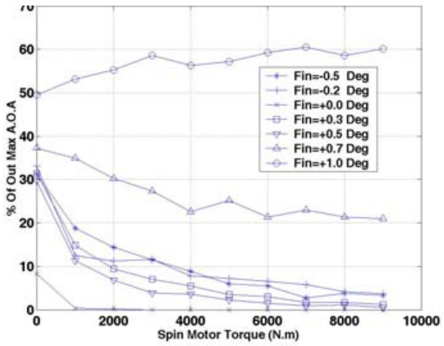
Fig 10: The freq. dist. of rocket Max. Vel. (M. Spin, Fin=0.5 deg.)



It should be pointed out that the aerodynamic model of the simulation software is valid over a certain range of the angle of attack and sideslip, therefore, any simulation run in which the maximum angle of attack or sideslip angle has exceeded the model validity range, can not be invoked. The generated data for these cases are only meaningless numerically calculated values and does not demonstrate the real flight specifications of the rocket. For a correct analysis, these kind of data must be removed from the total set of the output data. Similarly, there is a structural limitation on the maximum tolerable acceleration of the rocket. Again, those simulation runs showing unacceptable maximum values of longitudinal and lateral rocket accelerations are not useful and should be removed. In this analysis, acceptable maximum longitudinal and lateral accelerations are +/-20g and +/-10g, respectively [Saghafi and Khalilidelshad, 2003]. Also, the validity range of the aerodynamic model is up to 50 deg. angle of attack or sideslip.

The percentage of out of limit maximum angle of attack runs to the total runs for various combinations of fincant angle and spin motor torque are shown in Fig. 11. In general, the percentage of out of limit angle of attack is increased with increasing fincant angle and decreasing spin motor torque. It should be noted that flight in high angle of attack and load factor (high acceleration) as big as the given limiting values is practically impossible for an uncontrolled rocket. Therefore, flight-to-target in these cases are considered to be unsuccessful. Thus, the probability of flight-to-target success can be estimated by dividing the number of the out of limit simulation runs to the total number of runs.

Fig 11: The percentage of out of limit max. angle of attack



Having removed the unacceptable simulation data, the statistical characteristics such as mean values and standard deviations were calculated and used for examining the results. The variations of the mean value and standard deviation of the rocket directional impact point errors in different fincant angles versus spin motor torque are shown in Figs. 12-13. These results and the other similar plots for the rocket range and radial impact point errors, not presented here, show the great effect of the spin motor torque on the rocket impact point error reduction. Therefore, an obvious conclusion is that the maximum spin motor torque is the best value for all flight conditions.

Fig 12: The mean value of X-error in different launchs

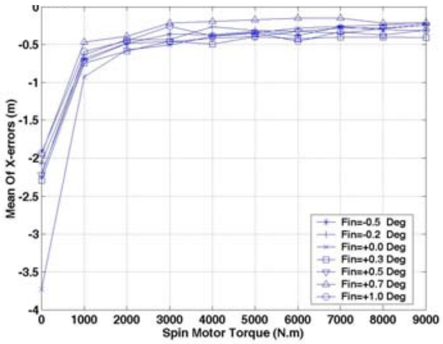
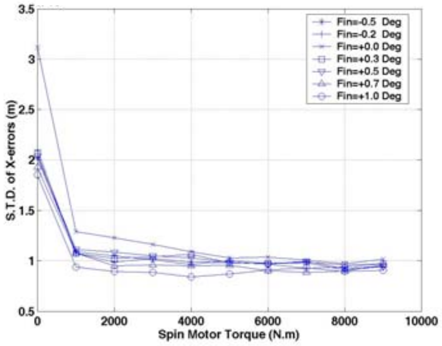


Fig 13: The standard deviation of X-error



Having set the torque, the fincant angle should be selected. A fincant angle corresponding to a minimum mean value and standard deviation of the impact point errors in maximum spin motor torque, would be the optimium value. To find this optimum value, dispersion plots for the range, directional and radial impact point errors in different fincant angles and maximum spin motor torque were used. Typical dispersion plots of these kinds for 0.3 and 1.0 deg. fincant angles are shown in Figs. 14-17. Using these plots, the cumulative probability of impact point errors to be in predefined limits, could be determined. The cumulative probabilities of the range, directional and radial impact point errors for different fincant angles and limitations are shown in Figs. 18-20. As shown, the cumulative probability does not change noticably with the fincant angles. In fact, there is no fincant angle which have a considerable effect on the cumulative probability of errors. Therefore, from the error point of view any fincant angle can be selected for the rocket. However, other considerations such as the severe effect of negative fincant angles on the rocket lateral acceleration, or fincant angles bigger than one, on impact point error, have limited the selectable values in the range of 0.0 to 1.0. Considering the possibility of manufacturing errors and to be far enough from the limits, a value of 0.5 deg. for fincant angle is proposed.

Fig 14: The probability distribution of X-error

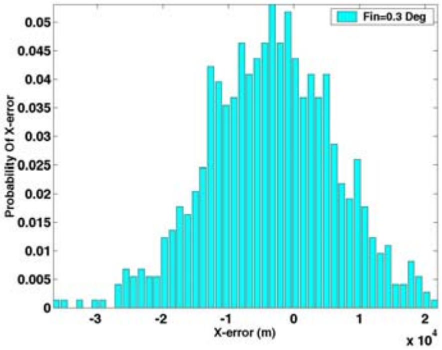


Fig 15: The probability distribution of X-error

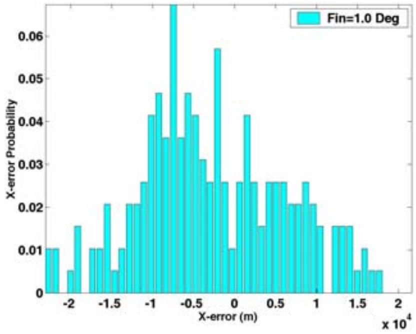


Fig 16: The probability distribution of radial error

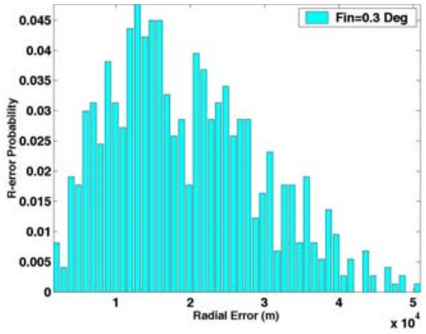


Fig 17: The probability distribution of radial error

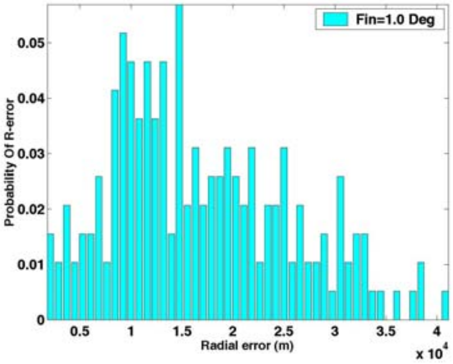


Fig 18: The cumulative probability of different ranges

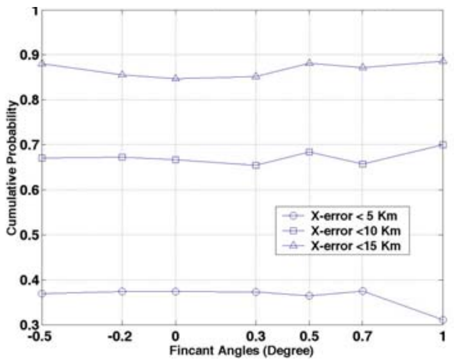


Fig 19: The cumulative probabilty of different Z-errors

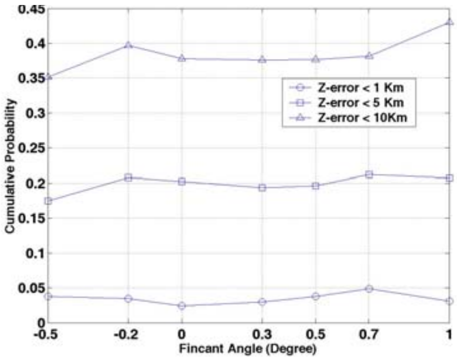
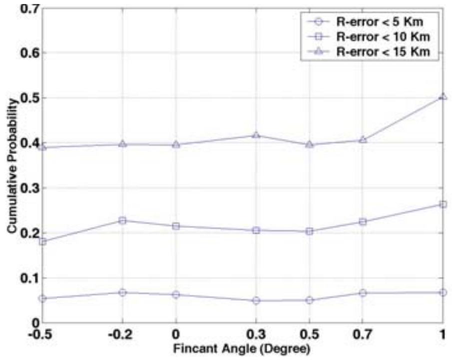


Fig 20: The cumulative probability of different radial errors



SUMMARY:

A Monte Carlo dispersion analysis of a medium range solid propellant simulation software was undertaken to show the usefulness of this type of analysis in the identification of design weaknesses in margins of specific parameters. Also, it is used to find out the optimum values of the rocket fincant angles and spin motor torque for the lowest impact point error, and the probability of flight-to-target success.

Results were presented for the selected conditions in the form of dispersion and statistical plots. This Monte Carlo analysis showed that the spin motor torque has a great effect on the rocket stability and its impact point error. Instead, the fincant angle has no noticable effect on these parameters in high values of spin motor torque. Finally, regarding other considerations, a fincant angle of 0.5 deg. together with a maximum value for spin motor torque are proposed.

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